AP Calculus Summer Assignment

For Parts A-E and G, refer to the notes and answers on the school website if you get stuck.
A. Piecewise Functions: (Do not use a calculator.)   Name_____

Graph the following piecewise functions:

1. $f(x) = \begin{cases} 5 & \text{if } x > 1 \\ 2 & \text{if } x \leq 1 \end{cases}$

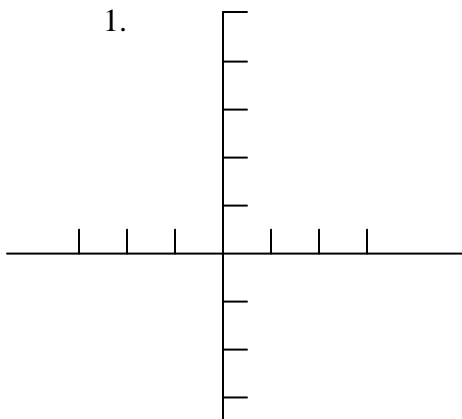2. $g(x) = \begin{cases} x^2 & \text{if } x \neq 1 \\ 3 & \text{if } x = 1 \end{cases}$

3. $f(x) = \begin{cases} x^2 & \text{if } x > 2 \\ 2x & \text{if } x < 2 \end{cases}$

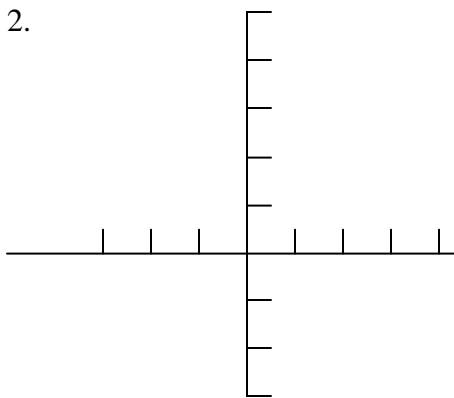4. $g(x) = \begin{cases} x^2 & \text{if } x > 1 \\ x & \text{if } x \leq 1 \end{cases}$

5. $f(x) = \begin{cases} 5 & \text{if } x > 0 \\ x+1 & \text{if } -2 \leq x \leq 0 \\ x^2 & \text{if } x < -2 \end{cases}$

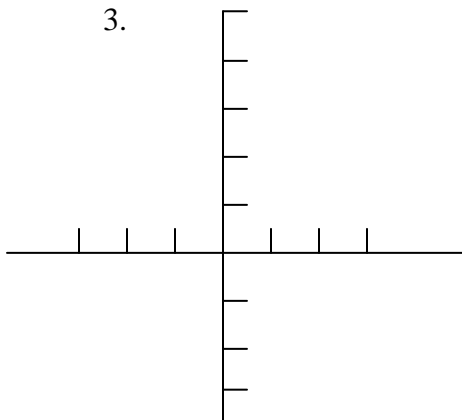6. $g(x) = \begin{cases} x+1 & \text{if } x < -1 \\ 3 & \text{if } -1 \leq x < 3 \\ x & \text{if } x > 3 \end{cases}$
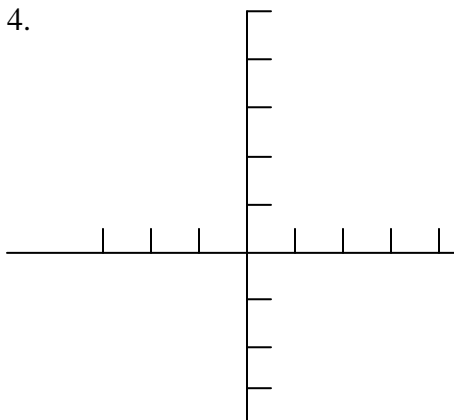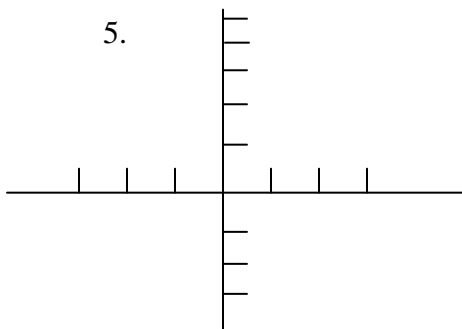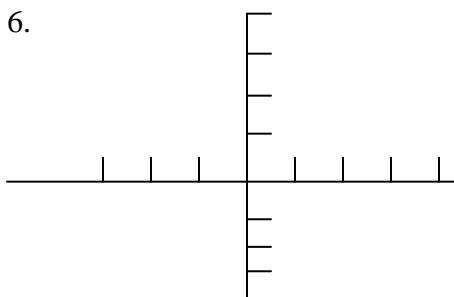
1.

2.

3.

4.

5.

6.

AP Calculus Summer Assignment

B. Interval Notation/Inequalities: (Do not use a calculator.)

Change from inequality notation to interval notation:
1. $-2 < x \leq 1$ 1_____
2. $x > 3$ and $x < 12.4$ 2 _____
3. $x \leq 3$ or $x \geq 10$ 3_____
4. All real numbers   4_____
5. All real numbers except 2   5_____
6. All real numbers except 2 and 4   6 _____
7. $x > 4$ and $x < 2$ 7_____
8. $x \leq 6$ and $x > -3$ 8_____

Change from interval notation to inequality notation: ($\cup$ -- or, $\cap$ - - and)
9. $(-3,4]$ _____

10. $(-\infty, 4) \cup [10, \infty)$ _____

11. $(-\infty, \infty)$ _____

12. $(-\infty, -2) \cup (-2, \infty)$ _____

13. $(-\infty, 4) \cap [3,5]$ _____

Solve the following inequalities. Give answer in both forms:

                                       Inequality form   Interval Notation

14. $x + 5 > 7$ _____   _____

15. $x + 7x - 8 \leq 0$ _____   _____

16. $x^2 + 7x - 8 \leq 0$ _____   _____

17. $x^2 - 3x - 4 > 0$ _____   _____

18. $x^2 - 3x + 4 > 0$ _____   _____

19. $x - 3x - 4 \geq 0$ _____   _____

20. $|x - 2| < 5$ _____   _____

21. $|x - 2| > 5$ _____   _____

22. $2(x - 3) + 4(x - 5) \geq 7x$ _____   _____

23. $\dfrac{5(x+2)}{(x-1)(x-3)} \geq 0$ _____   _____

C. Domain/Range : (Do not use a calculator. Skip the range in #5, 6, 8.)

Find the domain and range for the following functions:

	Domain	Range
1. $y = x^2 + 5$	_____	_____
2. $y = \sqrt{x+5}$	_____	_____
3. $y = \sqrt{x^2 - 9}$	_____	_____
4. $y = \dfrac{1}{x-3}$	_____	_____
5. $y = \dfrac{5}{x^2 + 3x - 4}$	_____	XXXXXXXXXXX
6. $y = \dfrac{5}{x^2 + 3x}$	_____	XXXXXXXXXXX
7. $y = \dfrac{x-5}{x-3}$	_____	_____
8. $y = \dfrac{5}{x^2 + 3}$	_____	XXXXXXXXXXX
9. $y = \sqrt{x^2 - 3x}$	_____	_____
10. $y = \log(x - 5)$	_____	_____
11. $y = e^{(x + 1)}$	_____	_____
12. $y = e^x + 1$	_____	_____
13. $y = 5\sin x$	_____	_____
14. $y = 5\sin x + 4$	_____	_____
15. $y = \tan x$	_____	_____

Give me a function whose domain is the following:

16. All real numbers except 2 _____   17. $x \geq 5$ _____

18. All real numbers except 2 and 3 _____ 19. $x > 5$ _____

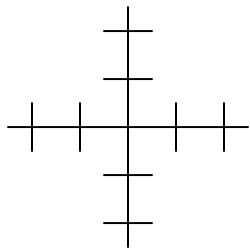Give the domain and range for the functions graphed below:

20.

D:_____ R:_____

21.

D:_____ R:_____
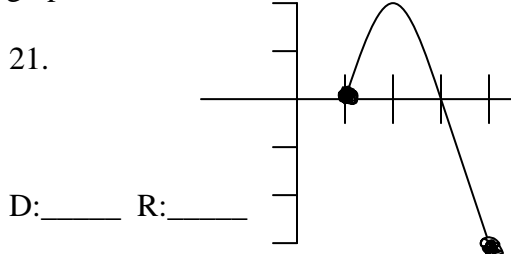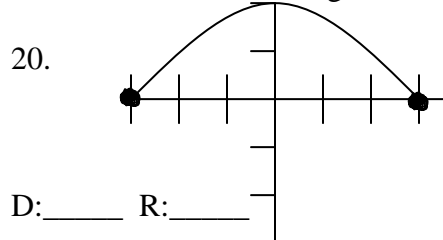
AP Calculus Summer Assignment

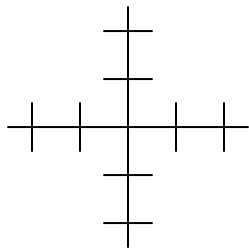D. Graphs of functions: (Do not use a calculator.)
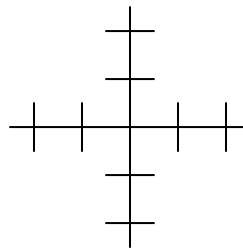
1. Sketch the following functions and label the vertex.

A. $y = |x| + 2$
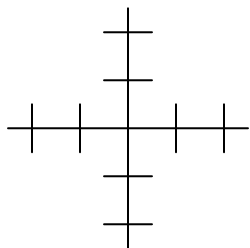
B. $y = -x^2 + 2$

C. $f(x) = (x-1)^3$

D. $y = -\sqrt{x+2} - 1$

E. $y = (x-2)^2 - 1$
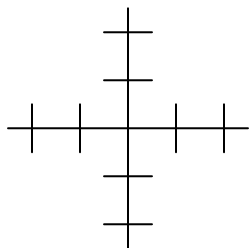
F. $f(x) = \sqrt{x-1}$
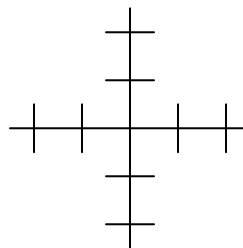
G. $y = -|x+2| + 1$
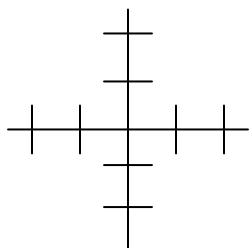
H. $y = -x^3 + 2$
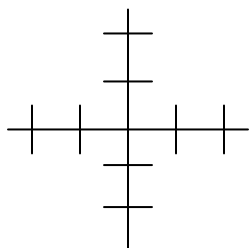
C. $f(x) = -2|x+1|$
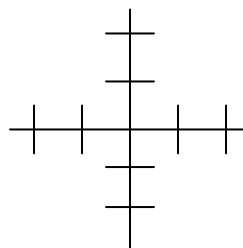
2. If f(x) is drawn to the right, then graph the following:

f(x)

A. f(x + 2)
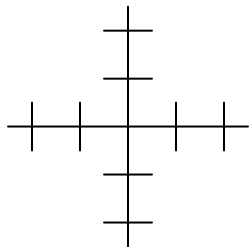
B. f(x) + 2
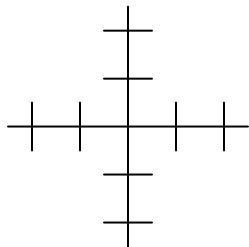
C. -f(x)
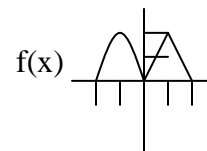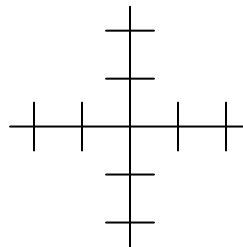
AP Calculus Summer Assignment

3. Graph the following:

A. $y = \dfrac{x^2 - 1}{x - 1}$

B. $y = \dfrac{|x - 1|}{x - 1}$
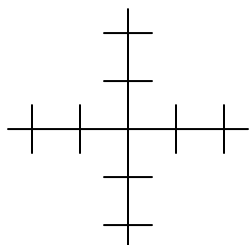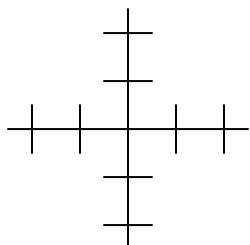
4. Find the domain/range and zeroes for:

A. $f(x) = [x]$ (Greatest integer function)   B. $y = \sqrt{3 - x}$

D: _____   D: _____

R: _____   R: _____

Z: _____   Z: _____

C. $f(x) = \begin{cases} x+1 & \text{if } x \leq 0 \\ 3 & \text{if } 0 < x \leq 1 \\ x^2 & \text{if } x > 1 \end{cases}$

D. $f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ -2 & \text{if } x > 1 \end{cases}$

D: _____   D: _____

R: _____   R: _____

Z: _____   Z: _____

AP Calculus Summer Assignment

E. Operations on Functions: (Do not use a calculator.)

1. If $f(x) = 2x - 3$, $g(x) = x^2 - 2$, and $i(x) = \sqrt{x} + 1$, then find:
   A. $f(2) =$ _____   B. $f(a+1) =$ _____   C. $i(4) =$ _____

   D. $g(3) =$ _____   E. $g(a+1) =$ _____   F. $i(2) =$ _____

   G. $f(g(2)) =$ _____   H. $g(i(9)) =$ _____

   I. $(f+g)(2) =$ _____   J. $fg(2) =$ _____

   K. $(f/g)(2) =$ _____   L. $(f-i)(1) =$ _____

   M. $g(g(x)) =$ _____   N. $f(f(x)) =$ _____

   O. $f(g(x)) =$ _____   P. $g(f(x)) =$ _____

   Q. $g(i(x)) =$ _____   R. $(f+g)(x) =$ _____

   S. $(f/g)(x) =$ _____   T. $f(g(i(4))) =$ _____

   U. $(f-g)(x) =$ _____   V. $f(g(i(x))) =$ _____

   W. $\dfrac{f(x+h)-f(x)}{h} =$ _____   X. $\dfrac{g(x+h)-g(x)}{h} =$ _____

2. If $f(x) = \dfrac{1}{x+2}$, $g(x) = \dfrac{4}{x-3}$, and $h(x) = 7x + 4$, then find:

   A. $(f+g)(x) =$ _____   B. $f(g(x)) =$ _____

   C. $f(h(x)) =$ _____   D. $(f-g)(x) =$ _____

   E. $f(g(h(-1))) =$ _____   F. $f(1/(g(2)) =$ _____

3. If $f(x) = 2x - 3$ and $g(x) = (x + 3)/2$, then find:

   A. $f(g(x)) =$ _____   B. $g(f(x)) =$ _____

   C. $f(g(7000)) =$ _____   D. $(f+g)(x) =$ _____

   E. $f(x)$ and $g(x)$ share a special relationship. These functions are _____ of each other.

4. If $f(x) = x^2 + 3x + 5$, then find:

   A. $\dfrac{f(x+h)-f(x)}{h} =$ _____   B. $\dfrac{f(3+h)-f(3)}{h} =$ _____

AP Calculus Summer Assignment

F. Trig Review: (Only use your calculator on #6, #7 and #11. All identities given below are not required to be memorized.)

Given Identities: $\sin(2x) = 2\sin x \cos x$ $\qquad$ $\cos(2x) = \cos^2 x - \sin^2 x$
$\sin(A+B) = \sin A \cos B + \sin B \cos A$ $\qquad$ $\sin(A - B) = \sin A \cos B - \sin B \cos A$
$\cos(A+B) = \cos A \cos B - \sin B \sin A$ $\qquad$ $\cos(A - B) = \cos A \cos B + \sin B \sin A$

1. Change $-\pi/4$ to degrees and find a positive coterminal angle.

2. If $\tan x = 3$ in quadrant I, then find the other trig functions: (Hint: Draw a triangle)

$\sin x =$ ____ $\qquad$ $\cos x =$ ____ $\qquad$ $\csc x =$ ____ $\qquad$ $\sec x =$ ____ $\qquad$ $\cot x =$ ____

3. Find the following values without a calculator: Draw the 30/60/90 and 45/45/90 triangles in the margin.

$\cos(\pi/3) =$ _____ $\qquad$ $\tan 45° =$ _____ $\qquad$ $\sin 60° =$ _____

4. Find cos 225° and show all work. (Show the angle drawn in the correct quadrant.)

5. Find 2 values of x on [0°, 360°], such that $\sin x = \frac{1}{2}$.

6. Find 2 values of x on [0°, 360°], such that $\cos x = 0.8123$ (Use your calculator)

7. Find the rest of the triangle if A = 40°, C = 90° and b = 6 (Use calculator)

a = ____ $\qquad$ A = ____

b = ____ $\qquad$ B = ____

c = ____ $\qquad$ C = ____
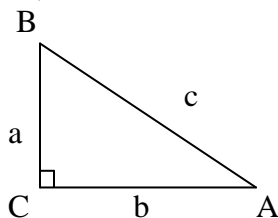
8. If $\cos x = -15/17$ and $\tan x < 0$, find csc x.

AP Calculus Summer Assignment

9. Find the following:
A. $\arccos(-\tfrac{1}{2}) =$ _____   B. $\sin(\arcsin(0.98)) =$ _____

C. $\tan(\sin^{-1}x) =$ _____   D. $\cos(\arcsin(12/13)) =$ _____

10. Fill in the following identities:
A. $\sin^2 x +$ _____ $= 1$ B. $\tan x = \sin x /$_____   C. $\csc x = 1/$_____

11. Evaluate $\cos 120°$ using a calculator.   _____

12. Evaluate $\cos 120°$ using its reference angle. Show your drawing:

13. In what 2 quadrants is cot negative? ___ ___

14. What is the difference between $\sin^{-1}$ and arcsin. Define them

15. Use trig identities to simplify the following trig expressions:
A. $\tan^2 x - \tan^2 x \sin^2 x =$ _____   B. $(\cos x - \sin x)^2 =$ _____

16. True or false: Explain.

A. $\sin x \bullet \csc y = 1$ _____   B. $3 \sec x = \dfrac{1}{3 \cos x}$ _____

17. Solve for values on $[0°, 360°]$ for $2\sin x + 1 = 0$

18. Solve $2\sin x + \sin(2x) = 0$ on $[0, 2\pi]$

19. If $\cos x = 5/13$ and $270° < x < 360°$, then find:
A. $\sin(2x) =$ _____   B. $\cos(2x) =$ _____

20. Find $\cos 15°$ exactly. (Hint $60 - 45 = 15$)

AP Calculus Summer Assignment

G. Simplifying Complex Fractions and Rational Expressions: (Do not use a calculator.)

Simplify the following (Find a common denominator in #10, 18-20)

1. $\dfrac{\frac{3}{x}}{y} =$ _____

2. $\dfrac{\frac{3}{x}}{\frac{y}{x}} =$ _____

3. $5x^{-2} =$ _____

4. $(5x)^{-2} =$ _____

5. $\dfrac{\frac{5}{x}+3}{y} =$ _____

6. $\left(9x^2\right)^{-\frac{1}{2}} =$ _____

7. $(\sin x)\bullet x + x^2 =$ _____

8. $(\sin x)\bullet x \bullet \cos x \bullet \sin x =$ _____

9. $\sqrt{x}\left(x + x^{-\frac{1}{2}}\right) =$ _____

10. $7x^{-\frac{1}{2}} + 3x^{\frac{1}{2}} =$ _____

11. $\dfrac{4x(2x-5)-(x^2-5x)\bullet(4)}{(2x-5)^2} =$ _____

12. $\dfrac{4(x-3)\bullet 2x - 5x\bullet\left(\dfrac{1}{x+3}\right)+24x}{y} =$ _____

13. $(\sin x)(\cos x)(\sec x)\bullet 7 + x^2(\tan x)(\sec^2 x)\bullet 5x =$ _____

14. $x\bullet e^x \bullet \dfrac{1}{2}x^{-\frac{1}{2}} - \sqrt{x}\sin x\bullet\left(-3x^{\frac{1}{2}}\right) =$ _____

15. $\dfrac{\dfrac{7x}{y}+\dfrac{2}{a}}{x+3} =$ _____

16. $\dfrac{\dfrac{7x}{y}-\dfrac{5}{3x}}{10} =$ _____

17. $\dfrac{\dfrac{2x}{3}-y^{-1}}{8-\dfrac{2}{y}} =$ _____

18. $\dfrac{4x}{2x-5}-\dfrac{3}{x-2} =$ _____

19. $\dfrac{7}{a-3}-4 =$ _____

20. $\dfrac{5}{\sqrt{z+1}}+\sqrt{z+1} =$ _____

AP Calculus Summer Assignment

H. Graphing Calculator Exercises: (A graphing calculator is required.)

1. Enter the following in the calculator exactly as written. Write each in standard notation and give the answer to each. (Note the use of parenthesis in the graphing calculator.)

A. -3^2 =____

B. (-3)^2 =____

C. $\sqrt{}$ (1 + 3) =____

D $\sqrt{}$ (1) + 3 =____

E. 1/3 + 5 =____

F. 1/(3 + 5) =____

2. Add 1/25 + 3/64 in the calculator. (Enter them in as you see them with the divide key. Once you get the decimal, use the key under the MATH menu FRAC, which will change it to a fraction. On some graphing calculators, if you use 2nd and divide instead of divide, it will make it a fraction.)

decimal: _____   fraction: _____

3. Find sin 90 in both degrees and radians. (In the MODE menu, there is a place to change from degrees to radians. Just scroll down to the appropriate one and hit enter.)

sin 90 (in degrees) = _____   sin 90 (in radians) = _____

4. Graph y = x^2 + 1 with x on [-1,2] and y on [-5,5]. (Use the WINDOW key and set xmin = -1, xmax = 2 xscl = 1, ymin = -5, ymax = 5 and yscl = 1. Then, hit the Y= key and enter in the function. Press graph.)

Draw the picture here:

5. Get a table of values for the function y = x^2 - 3 starting at 1 and going to 2, skipping by 0.1 each time. (Hit the TABLESET button and enter in starting table= 1 and Δtable = 0.1. Then enter the function above with the Y= key and push 2nd and GRAPH, which should be the TABLE button.)

Make your chart here:

| x	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
y											

AP Calculus Summer Assignment

6. I want to know the 2 roots of $y = x^2 - 3$ to the nearest hundredths place. (Graph the function on the interval [-5,5] by [-5,5], by changing the window.) Use the zero feature: Once the graph is on the screen, select ZERO under the CALC (2ND and TRACE) menu. It is #2 on TI-83's. It will ask for a left bound and then a right bound and finally guess. Pick a value to the left and then to the right of the root. Lastly, put the cursor as close to the root as possible when prompted guess and press enter. (Remember the root is where the graph touches the x-axis and is the solution to the equation $x^2 - 3 = 0$. The calculator should have the word ZERO above your answer.) Give 3 places after the decimal point for accuracy.

Answers: _____ _____

7. Graph $y = \sin x$ on two different intervals by changing the WINDOW to the x by y given:

A. in radians on $[0,2\pi]$ by [-2,2], with XSCL = $\pi/2$ and YSCL = 1

B. in degrees on [-360,360] by [-2,2] with XSCL = 90 and YSCL = 1
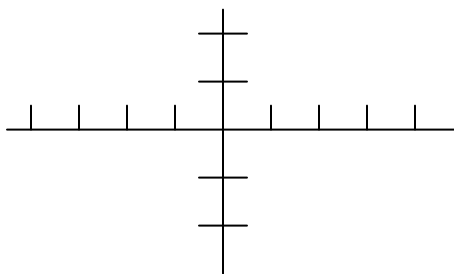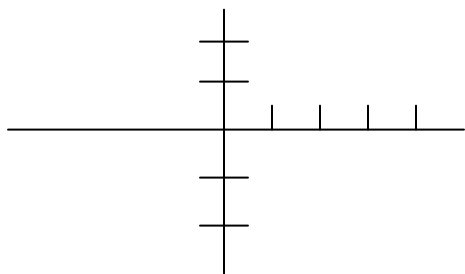
8. By graphing the following functions, give their domain and range:

*A. $y = \sqrt{9-9x^2}$ {Enter in as $\sqrt{}$ (9 -9x^2)}

B. $y = \sqrt{x-3} + 1$ {Enter $\sqrt{}$ (x-3)+1}

d: _____ r: _____

d: _____ r: _____

C. $y = \sqrt{3-x} + 1$

D. $y = x^2 - 4x + 5$

d: _____ r: _____

d: _____ r: _____

*For 8a, zoom in on the graph by adjusting the window until you see the semi-ellipse touch the x-axis twice. Use your knowledge of domain and range to check your work. No decimal answers.)

9. Find the roots of $y = x^2 - 16$ (Use the ZERO button)

_____

10. Find the roots of $y = x^3 + x^2 - 17$ (Use the ZERO button. Give 3 places after the decimal point for accuracy.)

_____

11. . Solve for at least one solution of each of the equations below using your calculator. All answers must be accurate to three decimal places and should include a quick sketch. Your calculators must be in radians. For the third one, get it equal to zero first. (Use the zero button. Give 3 places after the decimal point for accuracy.)

A. $x^7 - 11 = 0$ B. $\sin(x^2) + x - \cos x = 0$ C. $\cos x = \sin x + x$

_____   _____   _____

12. Graph the following functions and tell the difference the parenthesis makes in each problem. Rewrite the equations without parenthesis in standard notation:

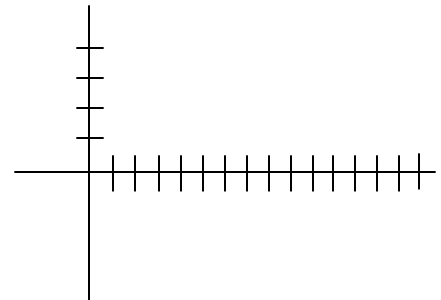   A. $y = 1/x + 2$ A_____

   B. $y = 1/(x + 2)$ B_____

13. The profile of a road in miles is given by P(x) below.

$$P(x) = \begin{cases} 3 + .04\cos(2x) & \text{if } 0 \le x \le 10 \\ \text{Bridge!!!!} & \text{if } 10 < x < 10.04 \\ 3.02 + 0.005(x - 11) & \text{if } x \ge 10.04 \end{cases}$$

   A. Sketch the road.

   B. What is the height of the bridge on each side?   _____ _____
           (Give 3 places after the decimal point for accuracy.)
   C. Approximate the slope of the bridge.
           (Give 3 places after the decimal point for accuracy.)   _____